Scale aspects of groundwater flow and transport systems

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Abstract Flow-system analysis is based on the concept of hierarchical groundwater flow systems. The topography of the water table, which is strongly related to the topography of the land surface, is a major factor in the hierarchical nesting of gravity-driven groundwater flow, resulting in flow systems of different orders of magnitude in lateral extent and depth of penetration. The concept of flow systems is extremely useful in the analysis of spatial and temporal scales and their mutual relationships. Basic equations on the laboratory scale are extended to larger, regional scales. Making use of Fourier analysis further develops Tóth's original idea of topography-driven flow systems. In this way, the different spatial scales of the water table are separated in a natural way, leading to a simple expression for the penetration depth of a flow system. This decomposition leads also to the relationship between spatial and temporal scales.

Analogous to flow systems, water bodies with different water quality may be called 'transport systems.' Field studies, numerical micro-scale modeling over macro-scale domains, and stochastic dispersion theory indicate that between systems with steady transport, the interfaces are relatively thin. The interfaces are much thinner than the relatively large mixing zones predicted by the conventional engineering approach to macrodispersion, in which relatively large, time-independent macrodispersion lengths are applied. A relatively simple alternative engineering approach is presented. For macrodispersion of propagating solute plumes, the alternative dispersion term gives the same results as the conventional engineering approach and gives correct results for steady-state transport.

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Résumé L'analyse des hydrosystèmes souterrains est basée sur le concept de systèmes hiérarchiques d'écoulement souterrain. La topographie de la surface piézométrique, qui est étroitement liée à celle de la surface du sol, est le facteur principal de l'emboîtement hiérarchique des écoulements souterrains, gouvernés par la gravité, ce qui fait apparaître des systèmes d'écoulement de différentes échelles en étendue et en profondeur de pénétration. Le concept de système d'écoulement est extrêmement utile pour analyser les échelles spatiales et temporelles et leurs relations mutuelles. Les équations de base correspondant à l'échelle du laboratoire sont étendues à des échelles régionales, plus vastes. L'utilisation de la méthode de Fourier met mieux en valeur l'idée originale de Tóth de systèmes d'écoulement commandés par la topographie. De cette façon, les différentes échelles spatiales de la nappe sont séparées naturellement, en donnant une expression simple pour la profondeur de pénétration du système d'écoulement souterrain. Cette décomposition fournit aussi la relation entre les échelles spatiale et temporelle.

Dans une approche analogue à celle des systèmes d'écoulement, des masses d'eaux de qualités différentes peuvent être appelées «systèmes de transport». Des études de terrain, une modélisation numérique à micro-échelle sur des domaines à macro-échelle et la théorie de la dispersion stochastique indiquent qu'entre des systèmes soumis à un transport en régime permanent, les interfaces sont relativement minces. Les interfaces sont beaucoup plus minces que les zones de mélange relativement étendues prédites par l'approche conventionnelle de l'ingénierie pour la macro-dispersion, dans laquelle on applique des longueurs de macro-dispersion, indépendant du temps et relativement étendues. Une approche d'ingénierie alternative, relativement simple, est présentée. Pour la macro-dispersion de la propagation de panaches de soluté, le terme alternatif de dispersion donne les mêmes résultats que l'approche d'ingénierie conventionnelle et donne des résultats corrects pour le transport en régime permanent.

Resumen El análisis de los sistemas de flujo se basa en el concepto de modelos jerárquicos de aguas subterráneas. La topografía del nivel freático, estrechamente relacionada con la topografía de superficie, es uno de los factores principales en la continuidad jerárquica del flujo subterráneo gravífico, dando lugar a sistemas de flujo con distintos órdenes de magnitud en lo que respecta a extensión lateral y profundidad. El concepto de sistemas de flujo es extremadamente útil para el análisis de las escalas espacial y temporal y de sus interrelaciones. Las ecuaciones básicas deducidas a escala de laboratorio se extienden a escalas regionales. Mediante análisis de Fourier se llega al esquema original de Tóth de sistemas de flujo dominados por la topografía. De esta manera, las diferentes escalas espaciales del nivel freático quedan separadas de manera natural, lo que conduce a una expresión simple para la profundidad de penetración en un sistema de flujo. Esta descomposición conduce además a una relación ente las escalas espacial y temporal.

De manera análoga a los sistemas de flujo, los cuerpos de agua de distinta calidad química pueden llamarse "sistemas de transporte". Tanto los estudios de campo como los modelos numéricos regionales con discretización a microescala, o la teoría estocástica de la dispersión indican que, para los sistemas con transporte estacionarios, las interfaces son bastante delgadas; más delgadas, por ejemplo, que las predichas por un tratamiento convencional de la macrodispersión, donde se utilizan valores relativamente grandes e independientes del tiempo. El estudio de la macrodispersión de penachos contaminantes se realiza mediante un modelo alternativo simple, donde el término alternativo de dispersión da los mismos resultados que los modelos convencionales.

Key words groundwater hydraulics · transport system · flow system · Fourier analysis · dispersion

Introduction

Flow system analysis is primarily based on the concept of hierarchical groundwater flow systems. Tóth (1962, 1963) first introduced this concept. In this approach, the topography of the water table, which is strongly related to the topography of the land surface, is considered as a major factor in the hierarchical nesting of gravity-driven groundwater flow patterns. This results in flow systems of different orders of magnitude in lateral extent and depth of penetration. This 'flow-systems concept' is extremely useful in many application fields, such as petroleum exploration (Tóth and Otto 1989; Verweij 1993), geo-environmental engineering (Engelen and Jones 1986; Engelen and Kloosterman 1996), and long-term geological waste disposal (Tóth and Sheng 1996). As shown in this paper, the concept of flow systems is also extremely useful in the analysis of spatial and temporal scales and their mutual relationships.

In this paper, the basic equations on the laboratory scale are extended to larger, regional scales. The original idea of topography-driven flow systems is further developed by making use of Fourier decomposition to separate the different spatial scales of the water table in a natural way. This approach leads to a simple expression for the penetration depth of a flow system as a function of the system's lateral extension and anisotropy of the subsurface. The relationship between spatial and temporal scales are explored by assessing the relation between the flow system's lateral extension, its penetration depth, and its characteristic time-scale for decay.

The temptation exists to apply groundwater flow system analysis to water-quality problems. Analogous with 'flow systems,' water bodies with different water qualities may be called 'transport systems.' Field studies (Sudicky 1983, 1986) and numerical micro-scale modeling over large, macro-scale domains (Frind et al. 1987) dealing with propagating plumes of solute dissolved in groundwater have both been reported in the literature. These studies indicate that steady flow systems that have been in existence for a sufficiently long time to make the transport of dissolved matter (quasi) steady have relatively thin mixing zones between the transport systems. The mixing zones are thin compared with the large macrodispersive mixing zones around a propagating solute plume. Such time-dependent behavior of macrodispersion can also be explained by stochastic dispersion theory (Dagan 1982, 1989). However, the existence of relatively sharp interfaces between steady transport systems *cannot* be predicted by the conventional engineering approach to macrodispersion modeling, in which relatively large, time-independent macrodispersion lengths are used in the transport model.

A relatively simple alternative to the conventional engineering approach is presented (simple compared with the highly sophisticated mathematics of stochastic dispersion theory). For the description of large-scale macrodispersion near propagating solute fronts, this alternative equation gives the same results as the conventional engineering approach. Moreover, it gives the correct results for steady large-scale solute transport. The value of this alternative approach is that it gives insight to the difference between unsteady and steady transport systems, without going into the intricacies of stochastic dispersion theory.

Basic Flow Equations and Parameters on the Fine Scale

Darcy's law $\underline{q}=\mu^{-1}\underline{\underline{\kappa}}\cdot\underline{e}$ or $q_i=\mu^{-1}\kappa_{ij}e_j$ relates the three components q_i, $i=x,y,z$, of the flux vector $\underline{q}$ [m.s^{-1} (m.d^{-1})] to the three components $e_j=-\partial\phi/\partial x_j$, $j=x,y,z$, of the driving force per unit volume $\underline{e}$ [N.m^{-3} (dbar.m^{-1})]; $\phi=p-\rho\underline{g}\cdot\underline{x}$ [Pa (dbar$=10^4$ Pa)] is the potential, p [Pa (dbar)] is the fluid pressure, ρ [kg.m^{-3}] is the fluid density (here assumed constant), $\underline{g}$ [on earth $|\underline{g}|$ = 10 N.kg^{-1} ($=10^{-3}$ dbar.m^2.kg^{-1})] is the gravitational acceleration, μ [Pa.s (dbar.d)] is the fluid viscosity and $\underline{x}$ [m] is the position vector. Fresh water has a specific weight of $\rho g = 10^4$ N.m^{-3} = 1 dbar.m^{-1}; in geohydrological practice, 1 dbar = 1 m (water column height) and $\rho g = 1$ disappears from the equations.

The tensor $\underline{\underline{\kappa}}$ [m^2] is the intrinsic permeability with nine components κ_{ij}, $i,j = x,y,z$; it is a property of the pore space only, independent of the fluid properties. The permeability generally differs from 'point' to 'point' (heterogeneity). It is practical to introduce the conductivity $\underline{\underline{k}} = \underline{\underline{\kappa}}/\mu$ [m^2.Pa^{-1}.s^{-1} (m^2.dbar^{-1}.d^{-1})]. On the fine scale, the conductivity tensor is symmetric, i.e., $k_{ij} = k_{ji}$ (King et al. 1995); however, after upscaling to larger volumes, this symmetry may be lost.

For incompressible (quasi-steady) flow, the continuity equation simplifies to div $\underline{q} = -\text{div}(\underline{\underline{\kappa}} \bullet \underline{\text{grad}}\ \phi) = 0$. When the principal directions of the conductivity tensor are horizontal and vertical, this equation simplifies to the Laplace-type equation $\partial/\partial x(k_h\ \partial\phi/\partial x) + \partial/\partial y(k_h\ \partial\phi/\partial y) + \partial/\partial z(k_v \partial\phi/\partial z) = 0$, where $k_h(x,y,z)$ and $k_v(x,y,z)$ are the horizontal and vertical conductivity, respectively.

Also, velocity-oriented Laplace-type equations can be derived. For instance, consider the quantities $e_x = q_x/k_h$, and q_z. For a *perfectly layered* subsurface, the Laplace-type equations are $\partial/\partial x(k_h\ \partial e_x/\partial x) + \partial/\partial y(k_h\ \partial e_x/\partial y) + \partial/\partial z(k_v\ \partial e_x/\partial z) = 0$ and $\partial/\partial x(k_v^{-1}\partial q_z/\partial x) + \partial/\partial y(k_v^{-1}\partial q_z/\partial y) + \partial/\partial z(k_h^{-1}\partial q_z/\partial z) = 0$ (Zijl and Nawalany 1993, pp 265–266). A perfectly layered subsurface is such that $k_h = k_h(z)$ and $k_v = k_v(z)$ are functions of the vertical z coordinate only. The existence of Laplace-type equations for velocity-oriented quantities shows that these quantities can be calculated directly, without making use of the potential. For transport problems, where flow velocities are the quantities of interest, it may be advantageous to numerically solve the velocity-oriented equations instead of the potential equation (Zijl and Nawalany 1993).

For a perfectly layered subsurface with discontinuous conductivities, the weak form of the equation for ϕ is that at discontinuity plane $z = d_i$, both ϕ and $k_v\ \partial\phi/\partial z$ are continuous. Similarly, the weak form of the equation for q_z is that at discontinuity plane $z = d_i$, both q_z, and $k_h^{-1}\ \partial q_z/\partial z$ are continuous. The weak forms are used in the next section to derive upscaled flow parameters.

Upscaling: Equations and Parameters on Regional Scales

In many cases, the parameters of formations in situ are different from the parameters measured in the laboratory on cores. Hence, the laboratory-scale parameters cannot be used in models on regional scales. An obvious cause of such a parameter mismatch is heterogeneity on spatial scales that are much smaller than the dimensions of the computational cells in a model. Present-day and even future computers are too limited to apply numerical models with the extremely large number of grid cells necessary to represent all the fine-scale variability. To study plume evolution in heterogeneous media, Frind et al. (1987) present a fine-scale modeling study on a large macro-scale domain. However, for engineering purposes such studies are too demanding. Therefore, to avoid an excessive amount of grid cells, upscaling is required to find cell-scale parameters (effective parameters, composite parameters, and 'equivalent homogeneous' parameters).

In Darcy's law on the fine scale, conductivity varies from point to point, notably in fractured rocks in which the conductivity of the fractures may be many orders of magnitude larger than that in the intact rock (Barenblatt et al. 1990; Bear et al. 1993). A point-scale distribution may be obtained from geological insight, or it may be generated in a probabilistic way. To obtain cell-scale parameters from point-scale parameter distributions, many types of upscaling procedures have been developed (Bensoussan et al. 1978; Zijl and Nawalany 1993; Fabrie et al. 1995; King et al. 1995; Zijl 1996; Lassing 1996; Trykozko 1997c; Rijpsma and Zijl, in press).

Upscaling Based on the Dupuit Approximation

The weak forms presented above can simply be used to give insight into the flow behavior relevant to upscaling procedures. It follows from the condition for q_z at interface i (on $z = d_i$) between layer i and layer $i+1$ that $(k_h^{-1})_i\ (\partial q_z/\partial z)_i = (k_h^{-1})_{i+1}\ (\partial q_z/\partial z)_{i+1}$. Suppose that layer i is an aquifer and layer $i+1$ is an aquitard, $(k_h)_{i+1}/(k_h)_i \to 0$; then it follows that $(\partial q_z/\partial z)_{i+1} \to 0$ on the interface $z = d_i$. As a consequence, it follows from a Taylor series expansion that $(\partial q_z/\partial z)_{i+1} \approx 0$ (is small) in the aquitard, provided that the aquitard $i+1$ is sufficiently thin. In other words, the vertical component of the flux, q_z, hardly changes over the thickness of a sufficiently thin aquitard. It follows then from Darcy's law that the well known relation $q_z = -k_v\ (\phi_{i+1} - \phi_i)/(d_{i+1} - d_i)$ holds. This result can be extended to aquitards with varying conductivities. If the vertical conductivity $k_v(z)$ varies with depth, it follows from Darcy's law and from the approximation q_z = constant over the layer thickness, that

$$
\begin{aligned}
q_z &= -(\phi_{i+1} - \phi_i) / \int_{d_i}^{d_{i+1}} k_v(z)^{-1}\, \mathrm{d}z = \\
&= -<k_v(z)^{-1}>^{-1} (\phi_{i+1} - \phi_i)/(d_{i+1} - d_i) = \\
&= -K_v <\partial\phi/\partial z> = -K_v \partial <\phi>/\partial z
\end{aligned}
\tag{1}
$$

The brackets $< >$ mean averaging over the z direction, and K_v is the upscaled vertical conductivity of the aquitard. In other words, the upscaled vertical conductivity of a perfectly layered subsurface with horizontal layers is equal to the harmonic mean of the fine-scale conductivities of the layers.

In a similar way it can be shown from the weak forms that the potential ϕ and the horizontal components of the driving force e_x and e_y hardly vary over the thickness of a sufficiently thin aquifer. In other words, $\phi(x,y,z,t)=f(x,y,t)$ is assumed to be independent of z. This approximation is generally called the 'Dupuit approximation.' A vertically constant potential and horizontal driving force does not mean that the flux in an aquifer is only in the horizontal direction. A non-zero vertical flux component may exist that can be determined from the continuity equation (Zijl and Nawalany 1993, pp 69–83).

Deviations from Dupuit-Based Upscaling

A more elaborate analysis based on perturbation theory (Van Dyke 1975) leads to the following form of the upscaled Darcy's law (Zijl and Nawalany 1993; Rijpsma and Zijl 1998).

$$\begin{pmatrix} <q_x> \\ <q_y> \\ <q_z> \end{pmatrix} = \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \bullet \begin{pmatrix} <e_x> \\ <e_y> \\ <e_z> \end{pmatrix} \tag{2a}$$

$$\begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} = \begin{pmatrix} <k> & 0 & \dfrac{M_x}{<1/k>} \\ 0 & <k> & \dfrac{M_y}{<1/k>} \\ \dfrac{N_x}{<1/k>} & \dfrac{N_y}{<1/k>} & \dfrac{1}{<1/k>} \end{pmatrix} \tag{2b}$$

$$\begin{aligned} M_x &= <k \int_{z_1}^{z} \frac{\partial}{\partial x}\left(\frac{1}{k}\right) dz'> - <k> < \int_{z_1}^{z} \frac{\partial}{\partial x}\left(\frac{1}{k}\right) dz'> \\ M_y &= <k \int_{z_1}^{z} \frac{\partial}{\partial y}\left(\frac{1}{k}\right) dz'> - <k> < \int_{z_1}^{z} \frac{\partial}{\partial y}\left(\frac{1}{k}\right) dz'> \\ N_x &= < \frac{1}{k} \int_{z_2}^{z} \frac{\partial k}{\partial x} dz'> - <\frac{1}{k}> < \int_{z_2}^{z} \frac{\partial k}{\partial x} dz'> \\ N_y &= < \frac{1}{k} \int_{z_2}^{z} \frac{\partial k}{\partial y} dz'> - <\frac{1}{k}> < \int_{z_2}^{z} \frac{\partial k}{\partial y} dz'> \end{aligned} \tag{2c}$$

The above equations represent the upscaled form of the isotropic Darcy's law on the fine scale $\underline{q}=k\,\underline{e}$, in which the fine structure of the conductivity is *not perfectly layered*, but is given by $k(x,y,z)$. Here $z=z_1$ is the cell boundary on which the potential is specified and $z=z_2$ is the cell boundary on which the flux is specified. For non-perfectly layered media, the upscaled conductivity tensor depends on the boundary conditions on the cell's boundaries. Because in reality there are no boundaries in the subsurface, this 'boundary dependence' reflects the fact that *upscaled parameters do not only depend on the point-scale parameter distribution within the cell, but also on the parameter distribution in the neighborhood of the cell.*

In the above expressions, the symbol $< >$ denotes averaging over only the z dimension of the cell; averaging over the x and y dimensions has still to be performed. After averaging over the x and y dimensions, the non-diagonal terms in the conductivity matrix vanish if the point-scale conductivity is periodic and the cell has the dimensions of one or more times one period. However, when a trend exists (a deviation from periodicity), the non-diagonal terms are non-zero and the upscaled conductivity tensor may be non-symmetric. This result is in agreement with homogenization theory for periodic media (Bensoussan et al. 1978; Lassing 1996; Trykozko 1997c), in which it is proved that any periodic fine-scale conductivity distribution that has a symmetric fine-scale conductivity tensor always leads to a symmetric upscaled conductivity tensor.

Anisotropy as a Scale Effect

An important aspect of the tensor character of the conductivity is anisotropy. Only when *symmetric* conductivity tensors are considered, anisotropy means that there are three directions – the principal directions – in which Darcy's law can simply be written as $<q_1>=K_1 <e_1>$, $<q_2>=K_2 <e_2>$, $<q_3>=K_3 <e_3>$. For non-symmetric conductivity tensors, the anisotropy has a more complex character (Zijl and Nawalany 1993, pp 89–96; Zijl 1996). Hence, for symmetric conductivity tensors, the flow is described as if it were flowing in an isotropic medium; however, with different conductivities in the three principal directions. For fine-scale conductivities, the anisotropy is generally modest, a factor of 3 at most. However, after upscaling, the anisotropy may be large; factors of 10 or even 100 may be encountered. The practical consequences of anisotropy are often underestimated; cross-flow phenomena and channeling caused by anisotropy may play an important role in the explanation of migration paths of matter dissolved in groundwater and of hydrocarbons migrating from the source rock to the reservoir rock (Verweij 1993). It is therefore important to have available computer models that are able to handle all types of anisotropy. In contrast to finite-difference methods, both the conformal-nodal and the mixed-hybrid finite-element method are well suited to study anisotropy (Trykozko 1997a, 1997b, 1997c).

Flow Systems Related to Spatial Fourier Modes

What is considered to be 'regional' and 'local' is largely subjective since, in general, it depends upon the size of the region being examined. Therefore, it makes sense to speak of a hierarchy of sublocal, local, supralocal, subregional, regional, supraregional, etc., spatial scales. For such a hierarchy of scales, classical trend analysis is not very suitable; therefore, Fourier analysis is applied.

The Spectrum of Spatial Scales in the Water table

The basic principle of Fourier analysis is that any function of x and y can be represented by a double integral of Fourier modes (Rikitake et al. 1987, pp 3–19; Bervoets 1991)

$$h_f(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_{(x,y)}(\omega,\xi,t)\,d\omega\,d\xi \tag{3a}$$

where $H_{(x,y)}(\omega,\xi,t)$ is one of the Fourier components into which the water table $h_f(x,y,t)$ is decomposed

$$H_{(x,y)}(\omega,\xi,t) = A(\omega,\xi,t)\cos(\omega x+\xi y) + \\ + B(\omega,\xi,t)\sin(\omega x+\xi y) \tag{3b}$$

where ω and ξ [m^{-1}] are the wave-number components in, respectively, the x and y directions. For a water table $h_f(x,y,t)$ defined on a finite domain $0 \le x \le L_x$, $0 \le y \le L_y$, the integral simplifies to a sum over discrete values ω_j, ξ_j, $j=1,2,...,\infty$ (Rikitake et al. 1987, pp 3–8).

Only after separation in Fourier modes do the scales come into play, since each Fourier mode $H_{(x,y)}(\omega,\xi,t)$ has its own spatial scale $\lambda = 2\pi/\sqrt{(\omega^2+\xi^2)}$, where $\lambda[m]$ is the wavelength of the Fourier mode under consideration. One of the attractive features of a Fourier series is that averaging over a specific spatial 'window' with width D is, roughly speaking, equivalent to eliminating from the series all the Fourier modes with a wavelength less than 2 D. Since the groundwater flow problem is linear, the solutions can be determined separately for each Fourier component $H_{(x,y)}(\omega,\xi,t)$. Thereafter, all these solutions can be integrated, or summed, over the wave numbers ω and ξ to obtain the complete solution.

To obtain qualitative insight into the concepts underlying flow-systems analysis, it is useful to consider a homogeneous and anisotropic subsurface. The potential $\phi(x,y,0,t) = f(x,y,t) = \rho\ g\ h_f(x,y,t)$ on the horizontal top plane $z=0$ represents a projection of the spatial variations of the water table on this plane. [Note that on earth $\rho\,g = 1$ dbar.m^{-1}; therefore, in most textbooks, where 1 dbar = 1 m, $f(x,y,t) = h_f(x,y,t)$.] To obtain a clearer picture of one particular Fourier mode and the groundwater flow caused by it, it is preferable to define the following $x'y'$ coordinate system in the horizontal plane

$$x' = (\omega x + \xi y)/\sqrt{(\omega^2+\xi^2)} \tag{4a}$$

$$y' = (-\xi x + \omega y)/\sqrt{(\omega^2+\xi^2)} \tag{4b}$$

The $x'y'$ axes are obtained by rotating the original xy axes over an angle, in such a way that $F_{(x,y)}(\omega,\xi,t) = \rho\ g\ H_{(x,y)}(\omega,\xi,t)$ can be written as a function of only the coordinate x' and the time t, independent of the coordinate y'

$$F_{(x')}(\lambda,t) = \rho g\{A(\lambda,t)\cos(\omega' x') + B(\lambda,t)\sin(\omega' x')\} \tag{5}$$

where $\omega' = \sqrt{(\omega^2+\xi^2)} = 2\pi/\lambda$. From the above expressions, by differentiation and by applying Darcy's law, on the top plane $z=0$ the horizontal flux component that is caused by the Fourier component under consideration is given by

$$Q_{X'(x')}(\lambda,0,t) = \rho\ g\ \omega'\ K_h \times \\ \{A(\lambda,t)\sin(\omega' x') - B(\lambda,t)\cos(\omega' x')\} \tag{6}$$

Tóth (1962, 1963) obtained the solution for the flow field in the $x'z$ cross section (Freeze and Witherspoon 1966, 1967; Domenico 1972, pp 256–264). From the general solution for one Fourier mode, the solution for an infinitely deep basin is presented here as an example. For that case, the vertical flux component on the top plane is equal to

$$Q_{Z(x')}(\lambda,0,t) = \omega'\sqrt{(K_h K_v)} \times \\ \{A(\lambda,t)\cos(\omega' x') + B(\lambda,t)\sin(\omega' x')\} \tag{7}$$

Also, the magnitudes of the flux components decay exponentially with increasing depth z according to

$$Q_{X'(x')}(\lambda,z,t) = Q_{X'(x')}(\lambda,0,t)\exp\{-(2\pi z/\lambda)\sqrt{(K_h/K_v)}\} \tag{8a}$$

$$Q_{Z(x')}(\lambda,z,t) = Q_{Z(x')}(\lambda,0,t)\exp\{-(2\pi z/\lambda)\sqrt{(K_h/K_v)}\} \tag{8b}$$

Penetration Depths Related to Lateral Spatial Scales

At depth z, the flux components of each Fourier component are given by $Q_{I(x')}(\lambda,z,t) = Q_{I(x')}(\lambda,0,t)$ $\exp(-\beta)$, $I = X'$, Z, in which $\beta = (2\pi z/\lambda)\sqrt{(K_h/K_v)}$. If the depth is chosen so that $\beta = \pi/2$, a decrease in flux intensity occurs by a factor $\exp(-\pi/2) \approx 0.21$. If z is chosen to be deeper, so that $\beta = \pi$, a decrease in flux intensity occurs by a factor $\exp(-\pi) \approx 0.043$. For $\beta = 3\pi/2$, the decrease in flux intensity is approximately 0.01. And for $\beta = 2\pi$, the decrease in flux intensity is approximately 0.002. Somewhat dependent on the situation, a negligibly small vertical flux component exists for one of these values of β. An infinite depth does not need to be chosen; already a good approximation of the solution is obtained when the depth of the 'effectively impervious' base is chosen equal to z, in such a way that $\beta = 2\pi$. Therefore, it makes sense to define the penetration depth

$$\delta = \lambda\sqrt{(K_v/K_h)} \tag{9}$$

Indeed, the solution of the flow problem with an impervious base at infinite depth is only a little different from the solution of the flow problem with an effectively impervious base at finite depth δ. The penetration depth δ defined in this way is proportional to the wavelength λ of the spatial variation in the water table and the anisotropy ratio $\sqrt{(K_v/K_h)}$. The choice $\beta = 2\pi$ is not arbitrary but is based on the fact that a conductivity distribution different from K_v and K_h below $z = \delta$ has little influence on the flow, whereas such a conductivity disturbance above $z = \delta$ influences the flow considerably (Meekes 1997).

Figure 1 shows an illustration of the above-discussed theory. *Figure 1a* shows the relatively shallow effec-

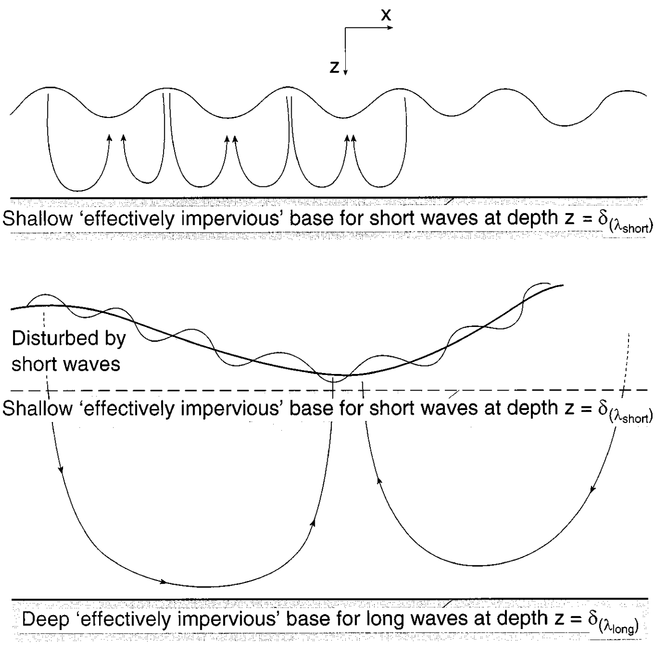


Figure 1 Penetration depths considered as 'effectively impervious' bases for short and long waves in the water table and the resulting streamline patterns

tively impervious base (the penetration depth) for the short waves in the water table. *Figure 1b* shows the deeper effectively impervious base for the long waves in the water table. Since the shallow flow velocities caused by the short waves have much larger flux intensity than the shallow flow velocities caused by the long waves, the shallow streamlines are almost fully determined by the short wavelengths. On the other hand, at depths below the effectively impervious base of the short waves, the flow velocities caused by the long waves are dominant. Hence, the deep streamlines are almost fully determined by the long waves.

Figure 2 shows the superposition of streamlines caused by a long, an intermediate, and a short wave in the water table. In this example, the three waves are assumed to be oriented in such a way that they cause flow in the same $x'z$ cross section. However, each Fourier component has its own $x'z$ cross section, which is generally rotated with respect to other cross sections. The dashed lines indicate streamlines that end at stagnation points (the black dots) in the flow field.

The concept of an effectively impervious base is more meaningful in a layered subsurface where aquifers and aquitards alternate. Indeed, in that case, the damping factor jumps abruptly in value, and it becomes clear that the effectively impervious base must be at the bottom of an aquifer and at the top of an aquitard. In that case, it is also possible to arrive at an exact solution for perfectly layered formations with piecewise constant conductivities (Bervoets 1991; Bervoets et al. 1991; Van Veldhuizen et al. 1993).

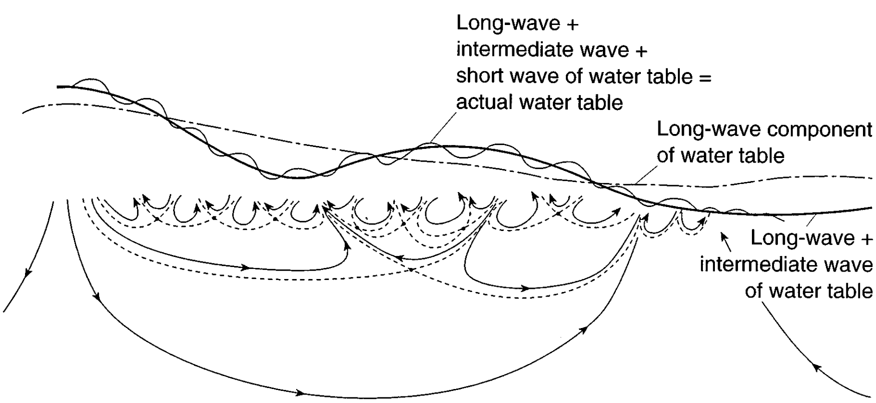
The Relation between Spatial and Temporal Scales

The supraregional spatial course of the water table is generally associated with the topography of the landscape. Tóth (1962, 1963) was the first to express this idea. Since the topography changes only slowly with time, the *deep* groundwater flow is almost steady, whereas the shallow groundwater flow is generally unsteady, because of the time-dependent recharge. This concept can be made more plausible by recognizing that the time dependence of the amplitude of a Fourier mode depends on the wavelength λ of that component. If, as an approximation, the kinematic boundary condition (phreatic storage condition) on the water table may be projected on the top plane $z=0$, and may also be linearized by neglecting the nonlinearities caused by the curvature of the water table, the superposition principle holds. Consequently, the projected and linearized kinematic boundary condition also holds for each Fourier mode separately

$$n\ \partial H_{(x')}(\lambda,t)/\partial t = P_{(x')}(\lambda,t) - Q_{Z(x')}(\lambda,0,t) \qquad (10)$$

where n is the phreatic storage coefficient, or specific yield S_y (or, approximately the porosity) and $P_{(x')}(\lambda,t)$ is the Fourier mode of the effective precipitation P. The effective precipitation has spatial variability, because it includes the effective infiltration at places where surface water occurs. (Therefore, a better name for P would be groundwater replenishment.) Now,

Figure 2 Superposition of long, intermediate, and short waves and the resulting streamline pattern



surface waters, such as rivers, canals, and ditches, often have a value of P that is controlled by surface-water flow in such a way that the water level remains approximately constant in time. Because of this overland flow, the effective infiltration, i.e., the values of P at the locations where surface water occurs has completely different values from those of the precipitation minus evaporation that occur elsewhere. Therefore, P has spatial variations that can be decomposed into Fourier modes, in which the wavelengths are determined by the orientation of, and the distances between, the surface waters.

It follows from Eq. (7) that in a homogeneous porous medium the following expression holds

$$Q_{Z(x')}(\lambda,0,t)=(2\pi/\lambda)\sqrt{(K_h K_v)}\,\rho g H_{(x')}(\lambda,t) \quad (11)$$

The above expression holds for incompressible (quasi-steady) flow, which means that elasto–plastic storage caused by compressibility of the water and the solid matrix has been neglected. Phreatic storage has been accounted for in the kinematic boundary condition. Combination with the kinematic water-table condition yields

$$n\,\partial H_{(x')}(\lambda,t)/\partial t=P_{(x')}(\lambda,t)-(2\pi/\lambda)\sqrt{(K_h K_v)}\,\rho g H_{(x')}(\lambda,t) \quad (12)$$

For the moment, assume that $P_{(x')}(\lambda,t)=P_{(x')}(\lambda)$ is constant in time. Then the water table asymptotically evolves to a steady state

$$H_{(x')}(\lambda,t)=H_{(x')}(\lambda,\infty)+\{H_{(x')}(\lambda,0)-H_{(x')}(\lambda,\infty)\}\times \exp[-\{2\pi/\lambda)\sqrt{(K_h K_v)}\,\rho g/n\}t] \quad (13)$$

where $H_{(x')}(\lambda,0)$ is the water-table height at $t=0$ and $H_{(x')}(\lambda,\infty)=P_{(x')}(\lambda)/\{(2\pi/\lambda)\sqrt{(K_h k_h)}\,\rho g\}$ is the steady water-table height that is asymptotically reached after long time. Hence, the characteristic time is equal to

$$\tau=n\,\lambda/\{2\pi\sqrt{(K_h K_v)}\,\rho g\} \quad (14)$$

From the above expression, the characteristic time becomes larger as the spatial extent of the variation in water table becomes greater. (Gravity also plays an important part, whereas in common geohydrological practice, where $\rho g=1$, this gravity effect is 'invisible.')

For instance, consider a regional wave with wavelength $\lambda=6.3$ km; and a subsurface with effective conductivities $K_h=1$ m^2.dbar^{-1}.d^{-1} and $K_v=0.0025$ m^2.dbar^{-1}.d^{-1}, and porosity $n=0.25$, the characteristic time $\tau=10{,}000$ d ≈ 27 yr. Hence, in this subsurface, flow at a depth greater than 315 m reacts to changes in natural groundwater recharge with a lag of approximately 27 yr. In practical terms, this means that this flow component is independent of the instantaneous effective precipitation, but is related to the topography and particularly to the surface waters on it. Because this flow component changes slowly, it can be charted on maps with a life span of, e.g., 20 yr. On the other hand, it follows that a relationship with the topography is much less likely for the local spatial course superimposed on the supraregional and regional courses. These short-wave water tables fluctuate in time with the seasons, or even on a daily basis (with an additional time lag due to flow in the unsaturated zone). For instance, for a local wave with a wavelength of 63 m and a subsurface with $K_h=1$ m^2.dbar^{-1}.d^{-1}, $K_v=0.1$ m^2.dbar^{-1}.d^{-1} and $n=0.35$, the characteristic time $\tau\approx 10$ d. (The ratio K_h/K_v is generally reduced with shorter, shallower waves.) So these short waves (with a depth of approximately 20 m) respond almost immediately to the seasonal, or even daily, fluctuations in recharge P.

Dispersion in Steady Transport Systems

The Engineering Approach to Macrodispersion

Numerical models for geohydrological engineering studies on the field scale, like studies on aquifer remediation or hydraulic isolation, are generally based on the conventional large-scale advection–dispersion equation (ADE) (Bear 1972)

$$\begin{aligned} n\,\partial\langle c\rangle\partial t+\mathrm{div}(\langle c\rangle\langle \underline{q}\rangle)= {}& \mathrm{div}[n\,\underline{\underline{D}}\bullet\underline{\mathrm{grad}}\langle c\rangle+ \\ & \mathrm{div}[|\langle \underline{q}\rangle|^{-1}\{A_L(\langle \underline{q}\rangle\bullet\underline{\mathrm{grad}}\langle c\rangle)\langle \underline{q}\rangle]+ \\ & \mathrm{div}[|\langle \underline{q}\rangle|^{-1}\{A_T(\langle \underline{q}\rangle\times\underline{\mathrm{grad}}\langle c\rangle)\times\langle \underline{q}\rangle\}] \end{aligned} \quad (15)$$

where $\langle c\rangle$[kg.m^{-3}(mg.L^{-1})] is the large-scale averaged concentration of dissolved matter (volume-averaged or flux-averaged). The term with tensor $\underline{\underline{D}}$ [m^2 . s^{-1} (m^2.d^{-1})] describes microdispersion: 'Particles' of solute and solvent are in random motion driven by the thermal motion of the molecules and ions of the solute–solvent mixture. Moreover, flow around the grains of porous rock is tortuous; velocity differences occur in the flow paths due to flow-channel branching. These phenomena lead to 'enhanced diffusion,' generally called microdispersion.

Large-scale transport on the field scale can be described in a more-or-less similar way as microdispersion on the fine scale. The resulting 'enhanced microdispersion' is generally called *macrodispersion*. The two types of dispersion, microdispersion and macrodispersion, are described by an anisotropic dispersion tensor with components that are proportional to the groundwater velocity. In the above ADE, the terms with the longitudinal and transversal macrodispersion lengths, A_L and A_T, respectively, describe the macrodispersion. In general, anisotropy exists such that $A_T \ll A_L$. The main differences between microdispersion and macrodispersion are in the magnitudes of A_L, A_T, and their microdispersion counterparts α_L, α_T, and in the fact that the macrodispersion lengths A_L and A_T are essentially time dependent.

Typical orders of magnitude for microdispersion are: $\alpha_L >$ average grain diameter, e.g., $\alpha_L \approx 10$ mm. Transversal microdispersion is generally one or more orders of magnitude smaller, i.e., $\alpha_T \ll 1$ mm. Microdispersion plays a role in laboratory studies, but its effect is too small to have a practical meaning for field studies. Therefore, to simplify the following discussion, microdispersion is neglected by assuming that its effect is negligibly small on the regional field scales under consideration.

For unsteady macrodispersion near propagating solute plumes, typical orders of magnitude are $A_L \approx$ average dimension of heterogeneity ≈ 10 m, and $A_T < 0.1$ m or smaller, sometimes an order of magnitude smaller. [$A_T \approx 0.01$ m for the Borden plume, where 'transversal' refers to the vertical direction (Sudicky 1986).] See also the numerical fine-scale study over a large-scale domain by Frind et al. (1987).

Field studies (Sudicky 1983, 1986) and numerical micro-scale modeling over large macro-scale domains (Frind et al. 1987) both show that during plume development the macrodispersion lengths are time dependent, with an infinitely small asymptotic ($t \to \infty$) transversal macrodispersion length. Gelhar and Axness (1983) and Neuman et al. (1987) develop stochastic dispersion theory for asymptotic dispersion. Dagan (1982, 1989) examines not only asymptotic dispersion, but also its evolution in time (pre-asymptotic macrodispersion). From these studies, it is inferred that during steady transport the dispersive interfaces are determined by microdispersion only, and are, therefore, relatively thin.

In practical large-scale modeling studies for engineering purposes, time-dependent macrodispersion lengths cannot be accounted for. First, their evolution in time is generally unknown and, secondly, the usual numerical models are not equipped to introduce time-dependent dispersion lengths. Therefore, in practical engineering studies the macrodispersion lengths are generally chosen sufficiently large to describe correctly the macrodispersion of a moving solute plume (Fetter 1994; Spitz and Moreno 1996). This choice means that such an engineering approach overestimates the mixing zones in *steady* transport systems.

The Relevance of Steady Transport Systems

In flow systems that have been in existence for sufficiently long times, mass transport has become steady. A practical example of flow systems under steady transport conditions is given by hydraulically isolated waste-disposal sites, where the polluted water is permanently pumped away by horizontal pumping and injection wells, as is shown in *Figure 3* (Trykozko 1997b). As explained above, in engineering practice the macrodispersion lengths are chosen such that the ADE correctly predicts the relatively large mixing zones accompanying the advancing plume. However, as a consequence, the predicted steady-state mixing zones are too large.

Similarly, the conventional engineering approach to macrodispersion may not be applied to compute the (almost) steady temperature fields in the interior of the earth, or the (almost) steady water-quality patterns in stable groundwater flow systems (Verweij 1993; Engelen and Kloosterman 1996), as illustrated in *Figure 4*.

The Mass-Transfer Approach

A relatively simple approach is described below that shows the origins of the time-dependent character of macrodispersion. For that purpose, consider the following two coupled mass-balance equations (Bear et al. 1993; Zijl and Nawalany 1993; Fabrie et al. 1995)

$$N\partial c/\partial t + (1-\sigma) < \underline{q} > \bullet \, \underline{\text{grad}} c = h(c' - c) \qquad (16a)$$

$$(n-N)\partial c'/\partial t + \sigma < \underline{q} > \bullet \, \underline{\text{grad}} c' = h(c - c') \qquad (16b)$$

In this case, water with solute concentration c flows with a relatively high velocity $(1-\sigma) < \underline{q} > /N$ (σ is a small non-negative number) through well conducting 'channels', e.g., a network of connected fractures. Also, water with concentration c' flows with relatively low, or zero velocity $\sigma < \underline{q} > /(n-N)$ through poorly conducting 'lenses', e.g., blocks of intact rock. Here, n is the fine-scale porosity, e.g., the porosity of the intact rock and of the porous material filling the fractures; and N is the flow-effective porosity, e.g., the pore volume in the well conducting channels per unit bulk volume. For more details, see Zijl and Nawalany (1993), who discuss the simpler case of no flow in the poorly conducting lenses ($\sigma = 0$). The basic ideas underlying this concept are also described by Gillham et al. (1984) under the name 'advection–diffusion concept.' However, they do not use the mass-transfer coefficient approximation to work out their ideas quantitatively. In a similar way, Barenblatt et al. (1990) introduces a mass-transfer term to approximate the transient-pressure evolution in fractured porous media.

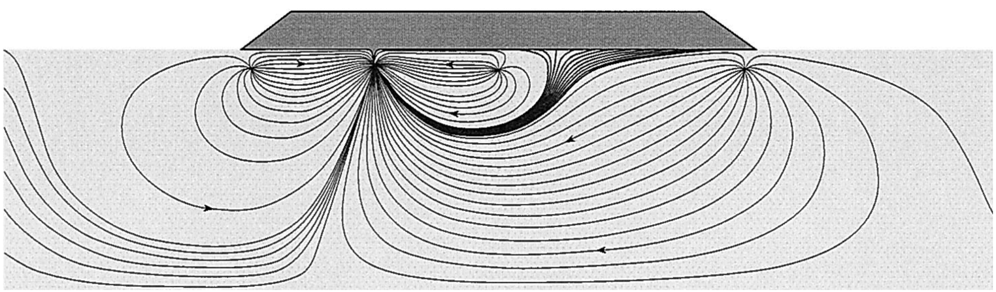


Figure 3 Waste disposal with horizontal pumping and injection wells for hydraulic isolation. Both groundwater flow and contaminant transport are steady. (After Trykozko 1997b)

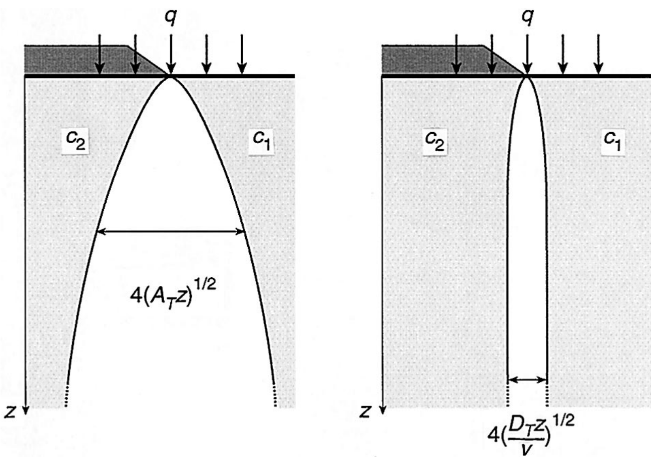


Figure 4a, b Zones with different water quality. Groundwater flow is steady. **a** The transition zone calculated with the conventional engineering approach to macrodispersion is much thicker than it is in reality **b** where it is caused by steady dispersion, which has the same order of magnitude as microdispersion ($A_T|<\underline{v}>| \gg D_T$)

In general, the mass-transfer coefficient h is proportional to a power of the Péclet number, the diffusion (here microdispersion) coefficient, and the inverse of a characteristic length (Bird et al. 1960, pp 642-649)

$$h = Pe^{\gamma}((n-N)/n)^2 D_T/B_L{}^2 \tag{17a}$$

where the Péclet number is defined as

$$Pe = |(n-N-n\sigma)/(n-N)|\,|<\underline{q}>|\,B_L/D_T \tag{17b}$$

D_T is the transversal microdispersion coefficient and B_L is related to the characteristic dimensions of the heterogeneities, e.g., the blocks of intact rock, over which upscaling takes place. The proportionality factors with n, N, and σ were chosen to simplify the derivations presented below. An approach based on the mass-transfer coefficient is not exact; rather, it is an approximation that can be justified only under a limited number of conditions, notably the condition of quasi-steady microdispersive transport from a well conducting channel to a poorly conducting lens.

Kunii and Suzuki (1967), who derived their theory in the context of heat and mass transfer in packed beds, show that, for $Pe<10$, $\gamma \to 1$. Using their result and combining the two mass-balance equations by eliminating c' yields an equation for c. Similarly, elimination of c yields exactly the same equation for c'. In this way, the equation for the volume-averaged concentration $<c>_V = \{Nc + (n-N)c'\}/n$, as well as the equation for the flux-averaged concentration $<c>_F = (1-\sigma)c + \sigma c'$ is given by

$$\partial<c>/\partial t + <\underline{v}>\bullet\underline{\mathrm{grad}}<c> = -(\eta^{-1}B_L/|<\underline{v}>|)D/Dt(\partial<c>/\partial t + <\underline{w}>\bullet\underline{\mathrm{grad}}<c>) = 0 \tag{18}$$

where $D(.)/Dt = \partial(.)/\partial t + [\sigma<\underline{q}>/(n-N)]\bullet\underline{\mathrm{grad}}(.)$ is the time derivative moving with the groundwater in the poorly conducting lenses; $<\underline{w}> = (1-\sigma)<\underline{q}>/N$ is the velocity in the well conducting channels; $<\underline{v}> = <\underline{q}>/n$ is the average velocity, and $\eta = |n-N-\sigma n|/N$. As is shown below, the last term of the above equation may be seen as an alternative dispersion term, in which time derivatives occur. Strack (1992) also obtained time derivatives in his dispersion term; however, he bases his derivation on assumptions that differ from the mass-transfer concept, or 'advection–diffusion concept' (Gillham et al. 1984) presented here.

When transport becomes steady ($\partial c/\partial t \to 0$)

$$<\underline{v}>\bullet\underline{\mathrm{grad}}<c> = E_L<\underline{v}>\bullet\underline{\mathrm{grad}}(<\underline{v}>\bullet\underline{\mathrm{grad}}<c>) \tag{19a}$$

$$E_L = -\sigma(1-\sigma)n^2N^{-1}(n-N)^{-1}\eta^{-1}B_L/|<\underline{v}>| \tag{19b}$$

For rocks in which flow in the lenses is negligibly small with respect to flow in the channels ($\sigma \to 0$), it follows that $<\underline{v}>\bullet\underline{\mathrm{grad}}<c> \to 0$ or, in other words, the concentration remains constant along the streamlines, without mixing zones. (For porous media where the velocity difference between channels and lenses is small, see the following section.) In reality, some mixing caused by microdispersion always exists (*Figure 4*), but on regional scales it can be neglected, meaning that the mass-transfer approach gives the correct behavior for steady-state transport.

For transport in the neighborhood of moving solute fronts, the alternative transport equation, with time derivatives in its dispersion term, can be transformed to the conventional ADE, without time derivatives in its dispersion term. The concentration $<c>$ is expanded in a perturbation power series of the small parameter $\varepsilon = A_L/(|<\underline{v}>|\tau)$, where $\tau \gg A_L/|<\underline{v}>|$ is a sufficiently large characteristic time scale for transport (Van Dyke 1975)

$$<c> = <c_0> + \varepsilon<c_1> + \varepsilon^2<c_2> + ... \tag{20}$$

Substitution of this perturbation series into the alternative transport equation, and equating terms with the same power of ε, shows that, up to first-order accuracy (up to $<c_0> + \varepsilon<c_1>$), this equation is equivalent with the following equation

$$\partial<c>/\partial t + <\underline{v}>\bullet\underline{\mathrm{grad}}<c> = (A_L/|<\underline{v}>|)<\underline{v}>\bullet\underline{\mathrm{grad}}(<\underline{v}>\bullet\underline{\mathrm{grad}}<c>) \tag{21a}$$

$$A_L = B_L[1-\sigma(1-\sigma)n^2N^{-1}(n-N)^{-1}\eta^{-1}] \tag{21b}$$

Again take the limit $\sigma \to 0$ (no flow in the lenses); then, $A_L = B_L$. Hence, in this case the alternative transport equation is equivalent to the conventional ADE (without transversal dispersion) with constant macrodispersion length A_L.

Thus, under conditions for which $\partial<c>/\partial t + <\underline{v}>\bullet\underline{\mathrm{grad}}<c> \approx 0$, i.e., in the neighborhood of a propagating concentration front, the conventional macrodispersion term without time derivatives, $(A_L/|<\underline{v}>|)<\underline{v}>\bullet\underline{\mathrm{grad}}(<\underline{v}>\bullet\underline{\mathrm{grad}}<c>)$ and the alternative macrodispersion term, with time derivatives, $-(\eta^{-1}$

$B_L/\ |<\underline{v}>|)D/Dt(\partial<c>/\partial t+<\underline{w}>\bullet\underline{\text{grad}}\ <c>)$, are equivalent.

The Relation Between Macrodispersion and Microdispersion

The previous discussion relates to rock matrices with poorly conducting lenses and well conducting channels, such as porous aquifers with clay lenses, or fractured rocks. In these cases, the flow-effective porosity N is sufficiently small with respect to the fine-scale porosity n to justify the 'mass-transfer approximation' as an alternative engineering description of macrodispersion. For homogeneous media without channels and lenses, this alternative transport equation, in which the macrodispersion term has time derivatives, reduces to the well established microdispersion equation, in which the microdispersion term has no time derivatives. For that purpose, consider porous media in the limiting case, where the poorly conducting lenses become infinitely small. In that case, the whole porous medium may be considered as a well conducting channel. In other words, consider the limit $N\rightarrow n$. At the same time, take the limit $\sigma\rightarrow 0$ in such a way that the same velocity $<\underline{q}>/n$ exists in the (infinitesimally small) lenses and in the channels $[\sigma\rightarrow(n\text{–}N)/n]$. Then, $B_L=-A_L|n-N-\sigma n|\rightarrow 0$. The meaning of $B_L\rightarrow 0$, $A_L\neq 0$ is that the terms with time derivatives in the alternative dispersion term vanish; only the time-independent part of this dispersion term remains. In this way, again the conventional ADE occurs

$$\partial<c>/\partial t+<\underline{v}>\bullet\underline{\text{grad}}<c>=\\ (A_L/|<\underline{v}>|)<\underline{v}>\bullet\underline{\text{grad}}(<\underline{v}>\bullet\underline{\text{grad}}<c>) \quad (22)$$

in which the dispersion term has no time derivatives. Hence, in homogeneous porous media the conventional ADE holds always, both near fronts *and in the steady state*. Moreover, in such porous media, the dispersion length A_L has the same order of magnitude as the microdispersion length α_L, which is already accounted for in the microdispersion tensor $\underline{\underline{D}}$.

Transversal Macrodispersion

Described above is longitudinal macrodispersion, which is dispersive transport in the direction of the upscaled flux $<\underline{q}>$. Transversal macrodispersion is a similar phenomenon; however, it occurs in directions normal to the upscaled flux. Transport normal to the upscaled flux $<\underline{q}>$ is caused by microdispersion, and its dispersive effect may be greatly enhanced by the fact that the fine-scale flux $\underline{q}$ has directions that deviate from the upscaled (volume-averaged) flux $<\underline{q}>$ (Zijl 1996). Since macrodispersion in the direction of $\underline{q}$ is exactly the same mechanism as macrodispersion in the direction of $<\underline{q}>$, it is reasonable to conclude that, in formations with well conducting channels surrounding poorly conducting lenses, transversal macrodispersion also disappears under steady-state transport conditions (*Figure 4*).

Summary and Conclusions

Flow-system analysis is mainly based on the concept of hierarchical groundwater flow systems, in which the topography of the water table, which is strongly related to the topography of the land surface, is a major factor in the hierarchical nesting of gravity-driven groundwater flow. The flow-system concept distinguishes flowing groundwater bodies with different orders of magnitude in lateral extent and penetration depth. This concept is very useful for the qualitative and quantitative understanding of groundwater flow-related phenomena in many application fields, such as petroleum exploration, geo-environmental engineering, and mining. Here it is shown that flow-system analysis is also extremely useful in the analysis of spatial and temporal scales and their mutual relationships.

Application of Fourier analysis has further developed the original idea of topography-driven flow systems. In this way, the different spatial scales can be distinguished in a natural way, leading to a simple expression for the penetration depth of a flow system. Also, the flow system's extension, its penetration depth, and its characteristic time have been related. Simple algebraic expressions, which provide much insight and are useful for order-of-magnitude estimations, are presented.

A temptation exists to apply groundwater flow-system analysis to water-quality problems. Analogous with flow systems, water bodies with different water quality may be called 'transport systems.' Field studies, numerical micro-scale modeling over large macro-scale domains, and stochastic dispersion theory indicate that in steady flow systems that have been in existence for a sufficiently long time to make the transport of dissolved matter steady, the interfaces between the transport systems are relatively thin, without large mixing zones. The presence of sharp interfaces contradicts the conventional engineering approach to dispersive transport and challenges the validity of flow- and transport-systems analysis, and even the validity of macrodispersion theory. A certain amount of mixing is always present, because of microdispersion. However, microdispersion is mainly a laboratory-scale effect that has no significant consequences for the much larger scales of practical field problems.

In practice-oriented studies on transport of solutes dissolved in groundwater, it is common practice to model large-scale dispersive transport in a similar way to the fine-scale microdispersive transport, *with relatively large*, *time-independent macrodispersion lengths*. In this paper, this is called the 'conventional engineering approach.'

In the analysis of water-quality flow systems (transport systems), one often deals with situations in which the transport has become steady. A practical example is a hydraulically isolated waste-disposal site. If the conventional engineering approach to large-scale macrodispersion theory were true, relatively large mixing zones would develop between the different transport systems. This would, for instance, invalidate the proper working of hydraulic isolation. However, it has often been observed in the field that the water-quality interfaces between transport systems are relatively sharp, almost without a mixing zone, which contradicts the conventional engineering approach. Also, numerical micro-scale modeling over large macro-scale regions indicates that in steady flow systems that have been in existence for a sufficiently long time to make the transport of dissolved matter steady, the interfaces between the transport systems are relatively thin, without large mixing zones. This fact is also inferred from stochastic dispersion theory.

This paper presents a relatively simple alternative approach, i.e., simple with respect to the highly mathematical intricacies of stochastic dispersion theory. For steady and unsteady microdispersion, as well as unsteady macrodispersion near propagating solute fronts, this alternative approach gives the same results as the conventional engineering approach to macrodispersion. Moreover, in contrast to the conventional engineering approach, the alternative approach describes correctly (quasi) steady transport. This result provides an argument in favor of flow- and transport-systems analysis, without contradicting the basics of well established dispersion theory.

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